The Case for Knightian Uncertainty:
Rationality and Psychology in Macroeconomics and Finance Theory

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Frank Knight's Distinction Between Risk and "True Uncertainty"

- "True uncertainty" arises from change that cannot be fully foreseen with probabilistic rules and whose consequences for market outcomes, and thus payoffs from market participants' decisions, cannot be fully comprehended – even in hindsight.

- For Knight, recognizing such unforeseeable change is the key to understanding profit-seeking activity in real-world markets.

  > *if all changes [...] could be foreseen for an indefinite period in advance of their occurrence [...] profit or loss would not arise (Knight 1921, p. 198).*

- Disciplinary consensus: profit-seeking activity can be understood with models which exclude Knightian uncertainty on the part of economists.
  - Adhering to this consensus, economists specify models that represent with a stochastic process how aggregate outcomes unfold over time.

Consensus as Obstacle to Progress in Macroeconomics and Finance Theory

- Focus on two approaches: the rational expectations hypothesis (REH) and behavioral finance.
- The core ideas that propelled the emergence of these two approaches are essential to further progress.
- But, formalizations of these core ideas with models that dispense with Knight's true uncertainty,
  - Obfuscated economists' characterizations of rational and irrational forecasting and decision-making.
  - Have rendered the core ideas underpinning REH and behavioral-finance approaches inherently incompatible.
- Building models that rest on the synthesis of these two approaches' core ideas requires recognizing that economists face Knightian uncertainty.

A Prototype Intertemporal Model

- Intertemporal macroeconomic and finance models relate endogenous variables – for example, the inflation rate or the stock price – to the forecasts of their future values by the market (an aggregate of its participants).

- A simple model for the stock price: market participants bid this price, p_t, to the level that satisfies the following no-arbitrage condition:

$$p_t = \gamma\left(\mathcal{F}_t\left(d_{t+1}\right) + \mathcal{F}_t\left(p_{t+1}\right)\right) \quad \text{for } t = 1, 2, \ldots \tag{1}$$

- We specify the dividend process, which relates dividends to one fundamental factor, corporate earnings, which we denote by x_t:

$$d_t = b_t x_t + \varepsilon_{d,t}, \tag{2}$$

- log-earnings follow a random walk with time-varying drift coefficients $\{\mu_t\}_{t=1,2,..}$,

$$\Delta \log x_t = \mu_t + \varepsilon_{x,t}, \tag{3}$$

The Rational Expectations Hypothesis (REH)

- REH rests on the core premise of economic analysis:
  - a market participant is assumed be rational insofar as he relates forecasts of market outcomes underlying his profit-seeking (or other goal-oriented) decisions to some understanding of the process driving these outcomes.

- Muth's (1961) striking hypothesis:
  - An economist can formalize participants' rationality – diverse understandings of the process driving outcomes – with the economist's own understanding of this process.

  - Muth proposed that an economist represent participants' forecasts of future outcomes as consistent with the predictions of his own model of these outcomes.

Representing Stock Prices and Market's Forecasts Precisely

- Dividends

$$\mathcal{F}_t\left(d_{t+k}\right) = b\left(v\exp(\mu)\right)^k x_t, \quad k = 0, 1, 2, \ldots \tag{4}$$

  where $v = E_t \exp(\varepsilon_{x,t})$.
- The stock price

$$p_t = \gamma\left(E_t(d_{t+1}) + E_t(p_{t+1})\right), \tag{5}$$

  Standard arguments then imply that the stock-price movements are driven solely by earnings:

$$p_t = \theta x_t, \quad \text{where } \theta = b\frac{\gamma v\exp(\mu)}{1-\gamma v\exp(\mu)}, \quad t = 1, 2, \ldots \tag{6}$$

Remark. *Once an economist assumes that he does not face Knightian uncertainty, applying Muth's hypothesis transforms the no-arbitrage condition (1) into a representation of the specific value that the stock price, p_t, takes at each t.*

Singular Representation of Rational Understanding

- REH's application of Muth's hypothesis necessarily assumes that the conditional expectation of an economic model's stochastic specification represents precisely how every market participant understands and forecasts the stock price.

$$\mathcal{F}_t\left(p_{t+k}\right) = \mathcal{F}_t^i\left(p_{t+k}\right) = E_t(\theta x_{t+k}|x_t) = \theta v^k \exp(k\mu)x_t \qquad (7)$$

Remark *Thus, whenever an REH economist imposes consistency within a consensus-compliant model, he transforms Muth's "sensible" hypothesis into an unfounded assertion that his model represents precisely a singular rational understanding of the process.*

Remark. *Although there is a substantial diversity of REH models that the profession considers relevant at any one time, each of these models implies that there is no diversity in how rational market participants understand and forecast outcomes.*

Excluding Participants' Autonomy and Psychological Factors from Rational Decision-Making

$$p_t = \theta x_t, \quad \text{where } \theta = b\frac{\gamma v \exp(\mu)}{1 - \gamma v \exp(\mu)}, \quad t = 1, 2, \ldots$$

Remark. *θ is completely determined by the discount factor, the model's coefficients (b, μ), and the moments of its innovations, v .*

Applying Muth's hypothesis in a consensus-compliant model rules out

- any autonomous role for participants' forecasts in driving the stock price.
- As a result, REH represents a rational participant's forecasts of stock prices to be driven *solely* by earnings.

$$\mathcal{F}_t\left(p_{t+k}\right) = \mathcal{F}_t^i\left(p_{t+k}\right) = E_t(\theta x_{t+k} | x_t) = \theta v^k \exp(k\mu) x_t$$

Behavioral-Finance Models

- Applying Muth's hypothesis in a consensus-compliant model renders it inherently incompatible with the raison d'être of behavioral finance:
  - psychological and other non-fundamental factors significantly influence how participants forecast outcomes.

- Adhering to the disciplinary consensus, behavioral-finance theorists have formalized the impact of psychological considerations on participants' forecasts with stochastic specifications.
  - As a result, they have had no option but to represent participants' forecasts with inconsistent models.

Consensus-Compliant Representation of Market Sentiment

- Barberis et al. (1998): market sentiment (participants' optimism or pessimism) regarding the future course of stock prices plays a significant role in driving participants' forecasts.

1. We characterize the market as being optimistic if its time-t forecast of p_{t+1} exceeds the REH forecast in (7), that is,

$$\mathcal{F}_t\left(p_{t+1}\right) = \theta^{\mathrm{H}} x_t \ > \theta x_t \tag{8}$$

2. We characterize the market as being pessimistic if

$$\mathcal{F}_t\left(p_{t+1}\right) = \theta^{\mathrm{L}} x_t \ < \theta x_t \tag{9}$$

3. We also characterize the market's switches between optimism and pessimism with a two-state Markov transition matrix.

The Gross Irrationality of Behavioral-Finance Models

- Although a few lay observers would deny that psychology plays a role in how individuals make decisions, a remarkably large number of economists have continued to uphold REH as a principle of model building.

- Arguably, the reason for this reluctance to recognize the relevance of behavioral findings for understanding market participants' forecasts is that model-inconsistent representations of these forecasts presume that participants are not merely "less than fully rational," as the behavioral-finance theorists Barberis and Thaler (2003) put it, but that they are grossly irrational.

Let the market be in an optimistic state:

$$\theta_t = \theta^{\mathrm{H}} \quad \text{and} \quad \mathcal{F}_t\left(p_{t+1}\right) = \theta^{\mathrm{H}} x_t \tag{10}$$

Standard arguments imply that the model represents p_{t+1} with the following probability distribution:

$$p_{t+1} = \gamma\left(b + \left(\theta^{\mathrm{H}} - \theta^{\mathrm{L}}\right) p + \theta^{\mathrm{L}}\right) x_t \ + \eta_{t+1} \tag{11}$$

The representations in (10) and (11) imply the following stochastic specification of the market's forecast error:

$$p_{t+1} - \mathcal{F}_t\left(p_{t+1}\right) = \gamma\left(\theta^{\mathrm{H}} - \theta^{\mathrm{L}}\right)(p-1) x_t \ + \eta_{t+1} \tag{12}$$

- The forecast error is systematically biased, $E\left(p_{t+1} - \mathcal{F}_t\left(p_{t+1}\right)\right) \neq 0$ for all t, and it is correlated with earnings.

Remark. *Model-inconsistent representations assume that, time and again, market participants are grossly irrational, in the sense that they forego profit opportunities indefinitely.*

Reconciling Participants' Rationality with the Autonomy of Their Forecasts

- By constraining the acceptable class of models to those that rule out unforeseeable change by design,
  - the disciplinary consensus has thus led economists to consider Muth's hypothesis to be inherently incompatible with the behavioral evidence that psychological considerations play a significant role in how participants' forecast outcomes.

- We show how, by leaving his model open to unforeseeable change, an economist can represent the role of both fundamental factors and psychological considerations, such as market sentiment, in rational participants' forecasting.

Knightian Uncertainty Expectations

- But leaving an economist's model open to Knightian uncertainty renders the conditional expectation – which represents a consensus-compliant model's prediction of outcomes – undefined.

- In Frydman et al. (2018), we develop a novel mathematical framework – called the Knightian Uncertainty Expectations (KUE) hypothesis – that defines a model's predictions of outcomes resulting from a process that changes at times and in ways that cannot be represented ex ante with probabilistic rules.

- The KUE framework applies Muth's hypothesis to represents how a rational market participant understands and forecasts outcomes in terms of fundamental factors.

- In contrast to REH, KUE's representations of rational forecasting are compatible with both diversity of participants' forecasts and these forecasts' autonomous role in macroeconomics and finance theory.

- Herein lies the fundamental importance of Muth's hypothesis that consensus-compliant REH models have obscured.
  - In choosing a particular quantitative forecast underlying their demand and supply decisions, rational, profit-seeking participants rely on a variety of factors and methods, including the predictions of a multitude of economic models on offer,
  - as well as intuitive, psychological, and other non-fundamental considerations.

Opening Macroeconomics and Finance Models to Knightian Uncertainty

- We replace the constraint that the impact of earnings on dividends is the same – equal to constants, μ and b – at every point in time with a particularly simple set of ex ante qualitative conditions, which we refer to as baseline constraints:

$$b_t \in [b_-, b_+], \text{ and } 0 < b_- < b_+, \tag{13}$$

$$\mu_t \in [\mu_-, \mu_+]. \tag{14}$$

- These ex ante constraints neither impose conditions on how b_t and μ_t will unfold over time, nor specify a probabilistic rule regarding which value these coefficients will take within their respective intervals.

Knightian Uncertainty Intervals

- Constraining the impact of earnings on dividends, $\{b_t\}_{t=1,2,..}$ in (2), with the ex ante baseline constraint in (13) implies that future b_{t+k}, when viewed from time t, lie within time-varying intervals $I^b_{t:t+k}$, that is,

$$b_{t+k} \in I^b_{t:t+k}. \tag{15}$$

and

$$d_{t+k} \in I^d_{t:t+k} = [b_- x_{t+k} + \varepsilon_{dt+k}, b_+ x_{t+k} + \varepsilon_{dt+k}], \quad k = 1, 2, ... \tag{16}$$

where we refer to $I^d_{t:t+k}$ as the (time-t) KU interval for dividends (at $t + k$).

Risk and "True Uncertainty"

- These intervals represent both standard (probabilistic) risk and what Knight referred to as "true uncertainty."
  - "Risk": The variance of the innovation $\varepsilon_{d,t+k}$.
  - "True uncertainty": Represented with the intervals $I^b_{t:t+k}$ and $I^d_{t:t+k}$.
- KU intervals enable an economist to formalize his own Knightian uncertainty.
- Importantly, because these representations hypothesize *ex ante* bounds on unforeseeable change in the model's coefficients,
  - they underpin KUE's definition of an economic model's predictions under Knightian uncertainty.

Predictions and Representations of Market's Forecasts Under Knightian Uncertainty

- Dividends are expected to lie within the following interval:

$$UE_{f_t}\left(I^d_{t:t+k}\right) = x_t v^k \left[b_- \exp(k\mu_-), b_+ \exp(k\mu_+)\right], \quad k = 1, 2, \ldots \tag{17}$$

where $v = E_t \exp(\varepsilon_{x,t})$.

Remark. *Because a KUE model is open to unforeseeable change, it does not specify which value d_{t+k} is expected to take within the UE interval in (17). Thus, in contrast to its REH counterpart's point predictions, the KUE model does not represent the prediction of d_{t+k} with one precise value.*

- Applying Muth's hypothesis represents the market's forecasts of d_{t+1} – denoted by $\mathcal{F}_t\left(d_{t+1}\right)$ – as follows:

$$\mathcal{F}_t\left(d_{t+1}\right) = \phi_t x_t \tag{18}$$

where

$$\phi_t \in v^k \left[b_- \exp(k\mu_-), b_+ \exp(k\mu_+)\right]. \tag{19}$$

Reconciling the Autonomous Role of Market Participants' Forecasts with Their Rationality

- Applying Muth's hypothesis represents $\mathcal{F}_t(p_{t+1})$ as follows:

$$\mathcal{F}_t(p_{t+1}) = \psi_t x_t, \quad \text{where } \psi_t \in [L_\gamma, U_\gamma] \tag{20}$$

- From no-arbitrage condition

$$p_t = \gamma(\phi_t + \psi_t) x_t \tag{21}$$

- Change in the stock price:

$$\Delta p_t = \gamma(\Delta\phi_t + \Delta\psi_t) x_t + \gamma(\phi_{t-1} + \psi_{t-1}) \Delta x_t. \tag{22}$$

  - Autonomous effect of change in ϕ_t and ψ_t: $(\Delta\phi_t + \Delta\psi_t) x_t$.
  - Effect of change in earnings: $(\phi_{t-1} + \psi_{t-1}) \Delta x_t$.

Remark. *Imposing consistency in a model that recognizes that an economist faces Knightian uncertainty reconciles participants" rationality with an autonomous role for their forecasts in driving outcomes.*

Market Sentiment and Participants' Rationality

- In order to represent the rationality of participants' forecasts of dividends and prices, we restrict the coefficients (ϕ_{t-1}, ϕ_t) and (ψ_{t-1}, ψ_t) to lie within the respective model-implies intervals:

  $$\phi_{t-1} \in v\left[b_- \exp(\mu_-), b_+ \exp(\mu_+)\right] \text{ and } \phi_t \in v\left[b_- \exp(\mu_-), b_+ \exp(\mu_+)\right]$$

  and

  $$\psi_{t-1} \in [L_\gamma, U_\gamma] \text{ and } \psi_t \in [L_\gamma, U_\gamma]$$

- We represent market sentiment with the increases (decreases) of the coefficients lying within these model-implied intervals:
  - The market is represented to be optimistic if $\Delta\phi_t + \Delta\psi_t > 0$.
  - The market is represented to be pessimistic if $\Delta\phi_t + \Delta\psi_t < 0$.

Remark. *These specifications capture the market's optimism and pessimism and yet are compatible with participants' rationality.*

Contingent Predictions under Knightian Uncertainty

- Imposing consistency within a consensus-compliant model implies quantitative predictions of how outcomes, such as stock-prices, co-move with the model's exogenous variables such as earnings.
  - For example, (8) implies that the stock price and earnings co-move between any two adjacent periods in the past and over an indefinite future:

$$\Delta p_t = \theta \Delta x_t, \quad t = 1, 2, \ldots$$

- Recognizing that economists face Knightian uncertainty places such historically non-contingent predictions out of reach for economic analysis.

- Under Knightian uncertainty, model's predictions are contingent on the movement of earnings and participants' forecasts between particular historical periods.

- Under Knightian uncertainty, the change in the stock price depends on the relative size of the effects of change in earnings and the change in an autonomous components of participants' forecasts.

- Given the formalization of market sentiment, our KUE prototype generates two examples of such contingent predictions.
  - First, contingent on future earnings increasing and the market being optimistic, the model predicts that the stock price will appreciate.
  - Second, contingent on future earnings decreasing and the market being pessimistic, the model predicts that the stock price will fall.

Remark. *Recognizing that economists face Knightian uncertainty opens a new path to understanding the role of market sentiment and other non-fundamental factors: these factors play a key role in generating an economic model's contingent predictions of co-movements in time-series data.*

Concluding Remarks

- The key question regarding the empirical relevance of Knightian uncertainty is how to ascertain whether the structural change is, at least in part, unforeseeable.

- The findings of a number of econometric studies point to the key reason why this is the case: structural change in models of outcomes, especially in financial markets, seems to occur contemporaneously with historical events that are not exact repetitions of similar events in the past.

    - Because of their uniqueness, these events give rise to change in the economy's structure that could not have been represented ex ante with probabilistic rules.

The Way Forward?

- Econometric evidence that consensus-compliant models encountered difficulties in predicting future outcomes in the past cannot decisively resolve the question of whether future research might discover the correct stochastic specification of the process driving outcomes.

- This persistent aspiration may help explain why, despite the oft-heard argument that, for example, REH-based Dynamic Stochastic General Equilibrium (DSGE) models "failed to predict" the 2008 crisis, many economists continue to believe that these models are useful in macroeconomic and policy analysis.

- Blanchard (2016) articulated a widespread position among adherents of REH-based macroeconomics.
  - He acknowledges that "the current DSGE models are seriously flawed,". However, he asserts that their flaws may make these models unsuitable for policy analysis. Indeed, he asserts that DSGE models "are *eminently improvable* and central to the future of macroeconomics" (p. 1, emphasis added).

Echoing some of Blanchard's main points, Gali (2018) provides an extensive survey of the REH-based New Keynesian models ten years after the 2008 financial crisis.

- He acknowledges, "Ten years later, tons of ammunition has been fired against modern macroeconomics in general, and against [DSGE] models that build on the New Keynesian framework in particular."

- However, underscoring the resilience of the disciplinary consensus, even after the 2008 crisis exposed its empirical difficulties, Gali points out what few professional observers would deny that:

> *The New Keynesian model arguably remains the dominant framework in the classroom, in academic research, and in policy modeling. In fact, one can argue that over the past ten years, the scope of New Keynesian economics has kept widening, by encompassing a growing number of phenomena that are analyzed using its basic framework (Gali, p.3).*